

# A new method for estimating the distance of young open clusters

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## ABSTRACT

We present a new technique for estimating the distance to young open clusters. The method requires accurate measurement of the axial rotation period of late-type members of the cluster: rotation periods are first combined with projected rotation velocities and an estimate of the angular diameter for each star - obtained using the Barnes-Evans relation between colour and surface brightness. A ‘best’ cluster distance estimate is then determined using standard techniques from the theory of order statistics, which are in common use in the general statistics literature. It is hoped that this new method will prove a useful adjunct to more traditional distance methods, in order to better ascertain the distance scale within the solar neighbourhood.

## Key words:

stars: distances – stars: late-type – stars: statistics – open clusters and associations: general.

## 1 INTRODUCTION

The accurate measurement of distances on intergalactic and extragalactic scales poses a problem of critical importance to both astronomical and cosmological research. There exists in common use a variety of different distance indicators; bridging and relating these indicators has enabled the construction of a cosmological distance ladder, the precise calibration of which is heavily reliant upon several key distances.

Of fundamental importance is the distance determination of nearby galactic clusters: for example the Hyades distance is a crucial ‘yardstick’ upon which the calibration of the extragalactic distance scale and ultimately the Hubble constant depend.

In this paper we describe a new technique for estimating the distance to nearby galactic clusters. The method is applicable to young clusters containing fast rotating late type stars, the periods of which may be determined by measuring rotational modulation due to surface inhomogeneities. Prime examples of such clusters are the Pleiades and  $\alpha$  Persei, which are the next step beyond the Hyades in the standard zero-age main sequence (ZAMS) fitting procedure.

The idea of using stellar rotation periods to estimate cluster distances was previously discussed in Collier Cameron & Woods (1992) (hereafter CCW) although no detailed statistical analysis was attempted. In CCW it was observed that the measured rotation period,  $P$ , of a star may be combined with the projected rotational veloc-

ity,  $v \sin i$ , to determine an estimate of the star’s ‘projected’ radius,  $R \sin i$ , viz:-

$$R \sin i = \frac{P v \sin i}{2\pi} \quad (1)$$

Here the inclination,  $i$ , takes its usual definition: the angle between the rotation axis of the star and the line of sight.

In CCW an estimate of the stellar angular diameter,  $\phi$ , was then inferred for each star using the relation between colour and surface brightness derived by Barnes, Evans & Moffett (1978) (hereafter BEM). (See also Section 2 below). A ‘projected’ cluster distance,  $D \sin i$ , then follows in the obvious way, viz:-

$$D \sin i = \frac{2R \sin i}{\phi} \quad (2)$$

where  $\phi$  is measured in radians. Substituting from equation (1) we find:-

$$D \sin i = \frac{P v \sin i}{\pi \phi} \quad (3)$$

Or, expressing in more convenient units:-

$$D \sin i = 7.660 \times 10^{-3} \frac{P v \sin i}{\phi} \quad (4)$$

where now  $P$  is measured in hours,  $v \sin i$  in  $\text{kms}^{-1}$ ,  $\phi$  in milliarcseconds and  $D \sin i$  in parsecs. This is essentially equation (4) of CCW.

$D \sin i$  values were thus derived in CCW for five stars in the  $\alpha$  Per cluster. The largest value of  $D \sin i$  was adopted

as a crude estimate of the cluster distance: this was based on the assumption that the largest observed  $D \sin i$  should correspond to the star with the highest axial inclination, so that  $\sin i$  might reasonably be taken to be equal to unity - the maximum value in the distribution of  $\sin i$ .

This simple approach did indeed give a cluster distance estimate which was in reasonable agreement, but greater than, the distance modulus  $(m - M)_o = 6.1$  quoted in Stauffer et al. (1985). The limitations of the approach, however, stem from the assumption that the distribution of  $D \sin i$  values arises solely from the intrinsic distribution of stellar inclinations, when in practice it results from a number of contributory factors: the intrinsic distribution of rotation periods, rotation velocities, inclinations and diameters - together with the observational scatter in the measurement of each of these quantities.

The aim of this paper is to extend and improve the simple treatment of CCW by addressing these limitations, and thus place the estimation of cluster distances from stellar rotation periods on a more rigorous statistical footing. In Section 2 we begin by describing in more detail the principles behind the calibration and use of the Barnes-Evans relation, upon which our distance method relies. In Section 3 we then carefully model the intrinsic scatter and observational selection effects which contribute to the distribution of  $D \sin i$  values. Such a detailed treatment is essential before one can meaningfully assign an error to any cluster distance estimate.

Having thus derived the  $D \sin i$  distribution expected for a given cluster, in Section 4 we consider several different distance estimators which one may define in terms of the observed  $D \sin i$  values. We investigate the properties of these estimators, demonstrating how one may assign the appropriate distance error to each, and how they may be used to construct confidence intervals for the true cluster distance. In particular we show how one may improve upon the crude estimator adopted in CCW by combining the  $D \sin i$  values inferred for *all* of the observed stars - instead of using simply the largest value. Finally, using artificially generated data, we examine how the accuracy of our new distance method varies with the number of observed cluster stars, and the intrinsic scatter in the Barnes-Evans relation. Based on these results we assess the usefulness of our new method set alongside the more traditional cluster distance indicators such as the ZAMS fitting procedure. In a forthcoming paper, currently in preparation, we will apply our distance method to a larger sample of  $\alpha$  Per stars and the Pleiades cluster for which rotation periods have been determined, and make a direct quantitative comparison between the cluster distance estimated by our method and that derived from ZAMS fitting.

## 2 THE RELATION OF BARNES AND EVANS (STELLAR ANGULAR DIAMETERS VS VISUAL SURFACE BRIGHTNESS/COLOUR)

In order to calculate a projected cluster distance,  $D \sin i$ , we have seen in Section 1 that it is necessary to estimate the angular diameter  $\phi$  of each star. This can be found to a limited degree of accuracy by introducing the work of Barnes & Evans (1976) (hereafter BE) who correlated stellar angu-

lar diameters to visual surface brightness and thus to colour index.

This correlation had been known for some time. Similar work was carried out by Wesselink (1969), Warner (1972) and Harwood et al. (1975) but was limited basically to early type stars and the B-V colour index. The work of Barnes and Evans extended the correlation to the entire UBVR system including red stars to spectral type M8. Their work was partly instigated by the sudden availability of many more stellar angular diameters found through a program observing the lunar occultations of nearby stars.

Barnes and Evans defined a quantity  $F_v$  the visual surface brightness parameter which can be shown to be:-

$$F_v = 4.2207 - 0.1V_o - 0.5 \log \phi \quad (5)$$

where  $V_o$  is the unreddened apparent magnitude and  $\phi$  is the stellar angular diameter expressed in milliseconds of arc.

Computing  $F_v$  for the nearby calibration stars and then plotting the resultant values against the various colour indices; B-V, V-R and R-I (shown in BE) confirmed a linear relationship between these indices and surface brightness. (A limitation was imposed for B-V where the relation breaks down and the index no longer relates to the energy output of the star). In BEM Barnes, Evans and Moffett improved these relations by adding more stars and correcting for the effects of limb darkening.

Of all the relationships the V-R vs  $F_v$  appears to have the tightest correlation but for the purposes of this paper we will follow the work of CCW and employ the B-V vs  $F_v$  relation which is linear over the spectral range of the  $\alpha$  Per G-K dwarfs.

From BEM the B-V vs  $F_v$  relation is linear over the interval:-

$$F_v = 3.964 - 0.333(B - V)_o, \quad -0.10 \leq (B - V)_o \leq 1.35 \quad (6)$$

where  $(B - V)_o$  is the unreddened colour index.

Equating equation (5) (theoretically derived) to equation (6) (empirically derived) and converting to reddened colour index and apparent magnitude gives the semi-empirical relation:-

$$\log \phi = 0.5134 - 0.066E_{B-V} + 0.666(B - V) - 0.2V \quad (7)$$

assuming  $A_v \simeq 3.0E_{B-V}$  for the Perseus region Hiltner & Johnson (1956) (see equation (3), CCW)

We can, therefore, obtain  $\phi$  from a knowledge of B-V,  $V$  and the colour excess  $E_{B-V}$  - for which an average value of 0.10 - 0.11 was adopted for the  $\alpha$  Per region (Crawford & Barnes (1974) and Prosser (1991)).

In effect we are working the original computation of Barnes and Evans backward to obtain  $\phi$  from  $F_v$  given colour and apparent magnitude  $V$ .

Most of the uncertainty in the stellar angular diameter  $\phi$  stems from the original lunar occultation measurements of the nearby calibration stars (typically 5 - 20 % (BE)). More difficult to quantify (due to the paucity of calibration stars) is the uncertainty in the interpolation of the  $F_v$  vs B-V graph within the range of our G-K dwarfs. Finally to a lesser degree we must also consider the systematic errors inherent from the measurement of B-V,  $V$  and  $E_{B-V}$  and the approximation  $A_v \simeq 3.0E_{B-V}$ .

The computed values of  $\phi$  generated from equation (6) are then substituted into equation (3) to obtain a  $D \sin i$

value for each star whose axial rotation period and projected equatorial velocity are known. Having generated a range of  $D \sin i$  values we then obtain a number of different cluster distance estimates by interpreting the ‘observed’ distribution of  $D \sin i$  values according to the statistical methods described in Section 4.

### 3 DISTRIBUTION OF DSINI

In this section we construct a model for the distribution of  $D \sin i$  values which one would expect to observe in a cluster at a given true distance. We will then later use this distribution as our basis for defining different cluster distance estimators and studying their properties.

A general method for deriving this distribution may be found in any elementary statistics textbook. Firstly one adopts a model for the joint probability distribution of the observed rotation period, rotation velocity, inclination and angular diameter - the variables in terms of which  $D \sin i$  is defined. This distribution must take account of the intrinsic spread in these variables from star to star, the scatter due to measurement error in the values observed for a given star, and any observational selection effects to which the sample may be subject. One next uses equation (3) to define an appropriate transformation of  $P$ ,  $v$ ,  $i$  and  $\phi$ , such that the distribution of  $D \sin i$  may then be extracted directly from the joint distribution of the transformed variables.

Although this method is straightforward to apply in principle, it rarely yields an analytic solution in practice, and that is indeed the case here. The problem is easily tackled via Monte Carlo simulations, however: i.e. we draw a large number of sets of the random variables,  $P$ ,  $v$ ,  $i$  and  $\phi$ , and for each set compute the corresponding value of  $D \sin i$ . We then deduce the distribution of  $D \sin i$  by constructing a histogram of the computed values.

Our analysis can be further simplified by making one additional assumption and a judicious change of variables, as we now show.

Suppose that we observe a cluster which lies at true distance  $D_{\text{true}}$  parsecs. We can regard  $D_{\text{true}}$  as a fixed - though of course unknown - parameter, which we wish to estimate. Consider a star in the cluster and let  $P_{\text{true}}$ ,  $v_{\text{true}}$  and  $\phi_{\text{true}}$  denote the true rotation period, rotation velocity and angular diameter (in radians) respectively of this star. Observe that these variables are not mutually independent, but are related by the following equation:-

$$D_{\text{true}} = \frac{P_{\text{true}} v_{\text{true}}}{\pi \phi_{\text{true}}} \quad (8)$$

Equation (8) assumes that the star lies precisely at the centre of the cluster - i.e. we neglect any line of sight depth.

Consider again equation (3) above, which expresses  $D \sin i$  in terms of the *observed* values of  $P$ ,  $v$ ,  $i$  and  $\phi$ . Introducing two new variables,  $z_p = \frac{P_{\text{obs}}}{P_{\text{true}}}$ , and  $z_\phi = \frac{\phi_{\text{obs}}}{\phi_{\text{true}}}$ , and combining with equations (3) and (8) we may write:-

$$D \sin i = \frac{D_{\text{true}} (v \sin i)_{\text{obs}} z_p}{v_{\text{true}} z_\phi} \quad \equiv \quad D_{\text{true}} \alpha \quad (9)$$

This change of variables has two immediate advantages. Firstly, we no longer require to model the *intrinsic* distribution of  $P$  and  $\phi$ , only the observational scatter about their

true values. One might reasonably expect these error distributions to be independent of the value of  $P_{\text{true}}$  and  $\phi_{\text{true}}$  respectively. Secondly, and more importantly, introducing  $z_p$  and  $z_\phi$  simplifies the dependence of the  $D \sin i$  distribution upon  $D_{\text{true}}$ . Clearly once we have modelled the distribution of the composite variable,  $\alpha$ , in equation (9), we can obtain the distribution of  $D \sin i$  for a cluster at *any* true distance simply by rescaling. One may think of  $\alpha$  as a dimensionless projection factor, essentially equivalent to  $\sin i$ .

As an example, Figures 1 and 2 show probability density curves for  $\alpha$ , determined for a particular selection function, and for different angular diameter error dispersions. The probability density curves were obtained by spline fitting to histograms constructed from 50000 trials. The Monte Carlo sampling was carried out as follows.

- (i) The true rotation velocity was first drawn from a uniform distribution in the range 0 to 240kms<sup>-1</sup>. For a star of one solar radius an equatorial velocity of 240kms<sup>-1</sup> corresponds to a rotational period of  $\sim 5$  hours, which was the shortest measured rotation period in the  $\alpha$  Per sample studied in O’Dell & Collier Cameron (1993).
- (ii) A true inclination was then assigned, based on the standard assumption that the orientation of the stellar rotation axis is completely random with no preferred direction in space (Bernacca (1970)). It follows easily from this assumption that the *intrinsic* distribution of  $\cos i$  is uniform over the interval [0, 1].
- (iii) The observed  $v \sin i$  was assigned by multiplying  $(v \sin i)_{\text{true}}$  by a Gaussian of unit mean and dispersion of 0.1; a relative error of 10% being typical for measurements of spectral line broadening of cluster stars (Stauffer et al. (1985)).
- (iv) A lower selection limit of  $(v \sin i)_{\text{obs}} \geq 50\text{kms}^{-1}$  was then imposed. This was the limit adopted in selecting the  $\alpha$  Per sample studied in O’Dell & Collier Cameron (1993). This selection function was primarily designed to ensure that only fast rotators were included in the sample - thus improving the chances of detecting rotational modulation within the available observing time.
- (v) Finally, the scaled variable,  $z_\phi$  was drawn from a gaussian of unit mean and constant dispersion,  $\sigma_\phi$ , where  $\sigma_\phi = 0.05$  in Figure 1, and  $\sigma_\phi = 0.1$  in Figure 2. Note that assigning a constant *percentage* error dispersion to the observed angular diameter was equivalent to a gaussian scatter of constant dispersion in the linear Barnes-Evans relation for  $\log \phi$ , as given by equation (7).
- (vi) In all cases the other scaled variable,  $z_p$ , was set identically equal to unity: i.e. it was assumed that the true axial rotation period of each star was recovered exactly from fourier analysis of the light curve. This approximation seemed reasonable when the data quality was sufficiently high to ensure little ambiguity in the star’s power spectrum. (See O’Dell & Collier Cameron (1993) for further discussion). In such a case the error in the derived period would be considerably smaller than the uncertainty in both  $\phi_{\text{obs}}$  and  $(v \sin i)_{\text{obs}}$ .

It is important to note at this point that the lower selection limit,  $(v \sin i)_{\text{obs}} \geq 50\text{kms}^{-1}$ , also serves indirectly to exclude stars of low inclination ( $i < 12^\circ$ ), since we are imposing an *upper* limit on the true rotation velocity. We would certainly expect some selection of this kind to exist in our

**Figure 1.** Probability density function of the scaled variable,  $\alpha$ , derived from a spline fit to the histogram obtained from 50000 Monte Carlo trials. The dispersion of the angular diameter errors was assumed to be  $\sigma_\phi = 0.05$ . The dashed curve indicates the intrinsic distribution of  $\sin i_{\text{true}}$  in the absence of observational errors and selection effects.

**Figure 2.** Probability density function of the scaled variable,  $\alpha$ , derived from a spline fit to the histogram obtained from 50000 Monte Carlo trials. The dispersion of the angular diameter errors was assumed to be  $\sigma_\phi = 0.1$ . The dashed curve indicates the intrinsic distribution of  $\sin i_{\text{true}}$  in the absence of observational errors and selection effects.

sample, since stars viewed at or near to pole-on would not display significant rotational modulation, and hence their periods could not be measured photometrically. The exact form of this selection at higher inclinations will depend upon the distribution of surface inhomogeneities, and in a more rigorous treatment this dependence could be modelled explicitly in our Monte Carlo sampling: i.e. for a given surface distribution we could determine the probability of detecting rotational modulation as a function of inclination. We do not incorporate such a model in this paper, however, since our aim is primarily to illustrate the essential features of our new distance method. Thus we adopt an inclination selection given simply by the limit of  $i < 12^\circ$ , as indicated above. In our next paper we will consider the precise form of the inclination selection in more detail - together with the inverse problem of how one might use our cluster distance

estimate to infer the distribution of surface inhomogeneities for the sampled stars.

We can see from equation (9) that, if one has perfect, selection-free, measurements of  $v \sin i$  and  $\phi$  then  $\alpha$  is identically distributed as  $\sin i_{\text{true}}$ . Hence, in this ideal case, the  $\alpha$  distribution rises monotonically from  $\alpha = 0$ , peaks at  $\alpha = 1$  and drops immediately to zero for all  $\alpha > 1$ , as shown by the dashed curves in Figures 1 and 2. Recall from section 1 that this behaviour motivated the choice in CCW of the largest  $D \sin i$  as the cluster distance estimate. When we include the effects of measurement errors and observational selection on  $v \sin i$  and  $\phi$ , however, we find that the *observed* distribution of  $\alpha$  - as indicated by the solid curves - is substantially different from the intrinsic distribution of  $\sin i_{\text{true}}$ , and this difference reveals a serious weakness in the CCW approach. Firstly we can see that the  $\alpha$  distribution no longer extends to  $\alpha = 0$  - i.e. the sample is biased against stars of low inclination, as discussed above. More importantly, however, the distribution displays a significant tail for  $\alpha > 1$ . Specifically, when  $\sigma_\phi = 0.05$  approximately 20% of the  $\alpha$  distribution lies in the range  $\alpha \geq 1$ . From equation (9), therefore, the probability equals  $\sim 0.2$  that one would infer - for any given star - a  $D \sin i$  greater than the true cluster distance. This probability increases to  $\sim 0.25$  for  $\sigma_\phi = 0.1$ . Moreover, for the sample of five stars considered in CCW, the probability that the *largest* value of  $D \sin i$  is greater than  $D_{\text{true}}$  increases to  $\sim 0.68$  for  $\sigma_\phi = 0.05$ , and to  $\sim 0.75$  for  $\sigma_\phi = 0.1$ . (These numbers follow easily from standard results on the distribution of *order statistics*, which we will introduce in Section 4, below). Thus we see that the cluster distance derived in CCW is quite likely an *over* - estimate, as a result of the failure to model the errors and selection on  $v \sin i$  and  $\phi$ . Note that the CCW estimate of the  $\alpha$  Per distance modulus was indeed somewhat larger than that quoted in e.g. Stauffer et al. (1985).

Can one define a better estimate of the cluster distance? Having shown how one may model the  $D \sin i$  distribution for any given cluster, we can now provide a quantitative answer to this question.

## 4 CLUSTER DISTANCE ESTIMATORS

Given a set of  $D \sin i$  values inferred from our sample of cluster stars we may define a number of different cluster distance estimators by combining these  $D \sin i$  values in various different ways. In this section we define and compare the properties of four such estimators, beginning with the estimator adopted in CCW.

### 4.1 ‘Naive’ Estimator, $\hat{D}_{\text{naive}}$

This estimator is simply the largest value of  $D \sin i$  in our sample, as discussed previously. Since we ignore the effects of errors in  $v \sin i$  and  $\phi$  in defining this estimator, we refer to it as ‘naive’. Suppose we observe a sample of  $n$  stars, and that we order the  $D \sin i$  values inferred for these stars in increasing size. Let  $\{D \sin i_{(k)} ; k = 1, n\}$  denote the ordered sample. i.e.:-

$$D \sin i_{(1)} \leq D \sin i_{(2)} \leq \dots \leq D \sin i_{(n)} \quad (10)$$

Then  $\hat{D}_{\text{naive}}$  is defined simply as:-

$$\hat{D}_{\text{naive}} = D \sin i_{(n)} \quad (11)$$

Note that we are adopting the usual statistical convention of using a caret to denote an estimator of a parameter. We will return to the statistical properties of ordered samples later in this section.

#### 4.2 ‘Mean’ Estimator, $\hat{D}_{\text{mean}}$

If we are to define a better distance estimator than  $\hat{D}_{\text{naive}}$  then clearly we must take some account of the true shape of the  $D \sin i$  distribution. One might also wish to make use of all the stars observed in the cluster, instead of just the star with the largest  $D \sin i$ . One obvious way to do this is by using the mean  $D \sin i$  of our sample.

Consider again equation (9) above. If we take the mean of both sides (more formally, the expectation value over the  $D \sin i$  and  $\alpha$  distributions at fixed true distance,  $D_{\text{true}}$ ), then it follows trivially that:-

$$\langle D \sin i \rangle = D_{\text{true}} \langle \alpha \rangle \quad (12)$$

We thus define our estimator,  $\hat{D}_{\text{mean}}$ , in terms of the sample mean value of  $D \sin i$  and  $\langle \alpha \rangle$ , the mean value of  $\alpha$  as determined from our modelled distribution function, viz:-

$$\hat{D}_{\text{mean}} = \frac{1}{n \langle \alpha \rangle} \sum_{k=1}^n D \sin i_{(k)} \quad (13)$$

An equivalent way of looking at this estimator is as follows: for each star individually we can derive a distance estimate, which is given by the cluster distance required so that the inferred  $D \sin i$  for that particular star is equal to the mean value of  $D \sin i$  at that distance.  $\hat{D}_{\text{mean}}$  is then simply equal to the mean value of these individual distance estimates.

#### 4.3 ‘Median’ Estimator, $\hat{D}_{\text{med}}$

We can derive another estimator based on the form of the  $\alpha$  distribution - this time using the median value,  $[\alpha]_{\text{med}}$ . Since this distribution is asymmetric, as we can see from Figures 1 and 2,  $[\alpha]_{\text{med}}$  will not generally equal the mean value,  $\langle \alpha \rangle$ . Our distance estimate is defined as the true distance for which the median of our ordered sample of observed  $D \sin i$  values is equal to the the median of the expected  $D \sin i$  distribution at that distance. Hence we have:-

$$\hat{D}_{\text{med}} = \frac{[D \sin i]_{\text{med}}}{[\alpha]_{\text{med}}} \quad (14)$$

where  $[\alpha]_{\text{med}}$  denotes the median of the modelled  $\alpha$  distribution and  $(D \sin i)_{\text{med}}$  denotes the median of the sampled  $D \sin i$  values. We can write down an expression for  $[D \sin i]_{\text{med}}$  in terms of the elements of our ordered sample, viz:-

$$[D \sin i]_{\text{med}} = \begin{cases} D \sin i_{(\frac{n+1}{2})} & n \text{ odd} \\ \frac{1}{2}(D \sin i_{(\frac{n}{2})} + D \sin i_{(\frac{n}{2}+1)}) & n \text{ even} \end{cases}$$

We can use a Monte Carlo approach to deduce the distribution of each of these estimators, and hence compare their relative accuracy. We generate a large number of artificial cluster samples of a given size - each sample drawn from our modelled  $D \sin i$  distribution for a cluster at some chosen true distance. For each sample we then compute the cluster

distance estimate predicted by each of the three estimators defined above. We determine the distribution of each estimator at the specified true distance by constructing a histogram of the computed values.

As an illustration, Figures 3 to 5 show a range of results obtained from generating 2000 cluster samples - containing 10, 20 and 30 stars, and for angular diameter error dispersions of 5% and 10% respectively. In all cases the true cluster distance was taken to be 200 pc, as indicated by the dashed line on each histogram. The mean values and dispersions of each estimator, as calculated directly from the histograms, are summarised in Table 1.

A consistent picture of the properties of the estimators emerges from these results. In all cases the naive estimator is positively *biased* - i.e. the mean value of the estimator is systematically greater than the true distance of 200 pc. One would expect, therefore,  $\hat{D}_{\text{naive}}$  to systematically overestimate the cluster distance, as was suggested in section (3). Contrary to what one might first expect, the magnitude of the bias of  $\hat{D}_{\text{naive}}$  in fact *increases* as the sample size increases: this is because, with a larger number of sampled stars, it becomes more likely that one will sample a star which lies further into the positive tail of the  $\alpha$  distribution. Hence  $\hat{D}_{\text{naive}}$  would become a progressively *poorer* distance estimator as we add more stars to our cluster sample. In the most extreme example considered here, for a cluster sample of 30 stars and for  $\sigma_\phi = 0.1$ , the mean value of  $\hat{D}_{\text{naive}}$  is more than 250 pc - representing a positive bias of  $\sim 25\%$ . It follows from the form of equation (9) that the percentage bias of  $\hat{D}_{\text{naive}}$  is independent of  $D_{\text{true}}$ , for a given sample size, so this figure would hold good for a cluster at any true distance. The bias of  $\hat{D}_{\text{naive}}$  also increases roughly in proportion to the angular diameter dispersion,  $\sigma_\phi$ , which is as one might expect.

The bias of  $\hat{D}_{\text{mean}}$  and  $\hat{D}_{\text{med}}$  on the other hand is negligible, irrespective of the sample size, for the cases considered here. This clearly demonstrates the importance of properly accounting for the true form of the  $D \sin i$  distribution in defining a good cluster distance estimate. From Table 1 we also see that the dispersion of  $\hat{D}_{\text{mean}}$  is found to be slightly smaller than that of  $\hat{D}_{\text{med}}$  in all cases, so that the former is marginally the more accurate estimator. We defer further discussion of Table 1 until later.

Having shown from our simulations that both  $\hat{D}_{\text{mean}}$  and  $\hat{D}_{\text{med}}$  are better distance estimators than  $\hat{D}_{\text{naive}}$ , we now consider whether one may improve still further upon the former two estimators.

In the case of a normal random variable, the sample mean is an example of a *sufficient* statistic (c.f. Mood & Graybill (1974)). Broadly speaking, this means that if one wishes to estimate the mean,  $\theta$ , of the normal distribution from a sample,  $\{x_i; i = 1, \dots, n\}$ , then the sample mean,  $\langle x \rangle = \frac{1}{n} \sum x_i$ , provides precisely the same statistical information about  $\theta$  as does the complete set of sampled values  $\{x_i\}$ . i.e. knowledge of each  $x_i$  would not improve our estimate of  $\theta$  compared with that obtained from knowledge of  $\langle x \rangle$  alone. This property does not hold in general for random variables which are not normally distributed, however. We can clearly see from Figures 1 and 2 that the distribution of  $\alpha$  is highly non-Gaussian: one might expect, therefore, that knowledge of the individual  $D \sin i$  values in our sample - and in particular their *ordering* - would allow one

**Figure 3.** Histograms of naive, mean and median cluster distance estimator distributions, derived from 2000 samples of 10 stars at a true distance of 200 pc. Results are shown for for an angular diameter error dispersion of  $\sigma_\phi = 0.05$  and 0.1

to define a ‘better’ (in the sense of having a smaller dispersion) distance estimator than  $\hat{D}_{\text{mean}}$  or  $\hat{D}_{\text{med}}$ . Before we introduce such an estimator we first make some preliminary remarks about the properties of order statistics.

The use of order statistics is a common technique in applied statistics, and the subject is treated extensively in a number of textbooks and monographs (c.f. David (1981), Gumbel (1958)). Suppose we draw a sample of size  $n$  from the distribution of our random variable,  $\alpha$ . By the  $r^{\text{th}}$  order statistic, which we denote by  $\alpha_{(r)}$ , we mean simply the  $r^{\text{th}}$  smallest member of the sample. Hence  $\alpha_{(n)}$  denotes the largest sampled value: this is precisely the notation introduced in equation (10) above.

The probability density distribution,  $p_r(\alpha_{(r)})$ , of the  $r^{\text{th}}$  order statistic is closely related to the distribution of the parent random variable,  $\alpha$ , viz:-

$$p_r(\alpha_{(r)}) = \frac{n!}{(r-1)!(n-r)!} [P(\alpha_{(r)})]^{r-1} [1-P(\alpha_{(r)})]^{n-r} p(\alpha_{(r)}) \quad (15)$$

where  $p(\alpha_{(r)})$  and  $P(\alpha_{(r)})$  denote respectively the probability density and cumulative distribution functions of the parent random variable,  $\alpha$ , in both cases evaluated at  $\alpha = \alpha_{(r)}$ . Figure 6 illustrates the distribution of several of the  $\alpha$  order statistics, for a sample of 10 stars drawn from the  $\alpha$  distribution shown in Figure 1 - i.e. for  $\sigma_\phi = 0.05$ . We can understand qualitatively how the shape of these distributions emerges from the form of equation (16). The distribution of  $\alpha_{(1)}$  for example has a larger dispersion than that of

**Figure 4.** Histograms of naive, mean and median cluster distance estimator distributions, derived from 2000 samples of 20 stars at a true distance of 200 pc. Results are shown for for an angular diameter error dispersion of  $\sigma_\phi = 0.05$  and 0.1

the higher order statistics  $\alpha_{(7)}$  and  $\alpha_{(10)}$  shown here. This is because the shape of  $p_1(\alpha_{(1)})$  is determined primarily by the behaviour of  $p(\alpha)$  at small  $\alpha$ , in which range the term  $[1 - P(\alpha)]$  in equation (16) is close to unity. We can see from Figure 1 that  $p(\alpha)$  is somewhat flatter in this range than for  $\alpha \simeq 1$ , and this behaviour is reflected in the shape of the higher and lower order statistics distributions.

Our aim is to use the properties of the  $\alpha_{(r)}$  distributions to define a better distance estimator. The question of how best to combine some or all of the ordered  $D \sin i$  values measured from our sample into one single distance estimate is non-trivial, however. One could, for example, derive a *separate* distance estimate - analogous to  $\hat{D}_{\text{mean}}$  or  $\hat{D}_{\text{med}}$  - from each ordered  $D \sin i$  in turn, and take the mean of these indi-

vidual estimates as our adopted cluster distance; the shapes of the order statistic distributions shown in Figure 6 suggest that it would be inappropriate to assign equal weight to each order, however.

Rather than, for example, adopting some *ad hoc* weighting scheme to resolve this problem, we construct an ‘ordered’ distance estimator,  $\hat{D}_{\text{ord}}$ , which combines the measured  $D \sin i$  values by a different - and rather more elegant - method: one which accounts naturally for the shape of each order statistic distribution in its definition.

**Figure 5.** Histograms of naive, mean and median cluster distance estimator distributions, derived from 2000 samples of 30 stars at a true distance of 200 pc. Results are shown for for an angular diameter error dispersion of  $\sigma_\phi = 0.05$  and 0.1

#### 4.4 ‘Ordered’ Estimator, $\hat{D}_{\text{ord}}$

Suppose we measure the value of  $D \sin i_{(r)}$ , for the  $r^{\text{th}}$  star in our ordered sample. Given this measured value, the probability distribution,  $p_r(\alpha_{(r)})$ , of the  $r^{\text{th}}$  order statistic of  $\alpha$  might now equivalently be regarded as a probability distribution,  $\lambda_r(D)$ , for the true cluster distance,  $D$ , viz:-

$$\lambda_r(D) \equiv p_r(\alpha_{(r)}) = \frac{D \sin i_{(r)}}{D} \quad (16)$$

This equation follows from equation (9) above.  $\lambda_r(D)$  is referred to as the *likelihood function* for  $D$ , given the measured value of  $D \sin i_{(r)}$ . Let  $\Lambda(D)$  denote the product of the likelihood functions for *all* orders,  $r = 1, \dots, n$ . viz:-

$$\Lambda(D) = \prod_{r=1}^n \lambda_r(D) \quad (17)$$

We define our ordered cluster distance estimate,  $\hat{D}_{\text{ord}}$ , as the value of  $D$  which maximises  $\Lambda(D)$ ; i.e.  $\hat{D}_{\text{ord}}$  satisfies:-

$$\left. \frac{\partial \Lambda}{\partial D} \right|_{D=\hat{D}_{\text{ord}}} = 0 \quad (18)$$

Since we do not have an analytic form for  $\Lambda(D)$  we cannot calculate  $\hat{D}_{\text{ord}}$  directly by differentiation. It is, nevertheless, straightforward to determine  $\hat{D}_{\text{ord}}$  simply by computing  $\Lambda(D)$  over a range of trial distances and finding the distance which yields the maximum value.

It is instructive to compare the properties of  $\hat{D}_{\text{ord}}$  with the other estimators which we have discussed, and discover



if it is indeed a ‘better’ estimator. We can do this again by using Monte Carlo simulations to construct the distribution of  $\hat{D}_{\text{ord}}$  at a given true distance.

As an illustration Figure 7 shows the results obtained from computing  $\hat{D}_{\text{ord}}$  for 2000 cluster samples at a true distance of 200 pc. As in Figures 3 to 5, sample sizes of 10, 20 and 30 stars and angular diameter errors of 5% and 10% are considered. A range of trial distances from 100 pc to 300 pc, at intervals of one pc, was tested. Given the spread in the distributions obtained, clearly a smaller step-size would have been redundant.

Figure 7 demonstrates that  $\hat{D}_{\text{ord}}$  is a slightly more accurate distance estimator than each of three estimators previously considered: in all cases the bias of  $\hat{D}_{\text{ord}}$  is found to be negligible, and the dispersion appreciably smaller than that of both  $\hat{D}_{\text{mean}}$  and  $\hat{D}_{\text{med}}$ . These results are also summarised in Table 1. We can see from Table 1 that the dispersion of  $\hat{D}_{\text{ord}}$  is around 10% smaller than that of  $\hat{D}_{\text{mean}}$  in all cases. The improvement gained by using  $\hat{D}_{\text{ord}}$  appears to be greater for smaller sample sizes: i.e. there is less difference between  $\hat{D}_{\text{ord}}$  and  $\hat{D}_{\text{mean}}$  for  $n = 30$  than for  $n = 10$ . This would seem to be consistent with the central limit theorem, which requires that  $\hat{D}_{\text{mean}}$  is asymptotically normally distributed (and hence a sufficient statistic) as  $n$  increases. It is also worth noting that the dispersion of all of the estimators decreases more noticeably when one increases the sample size from  $n = 10$  to  $n = 20$ , as compared with an increase from  $n = 20$  to  $n = 30$ . Further studies bear out this trend for larger samples: beyond a sample size of  $n \simeq 30$  there is very little further gain in the accuracy of the distance estimators - and in particular  $\hat{D}_{\text{ord}}$  - by adding new stars to the cluster sample.

The estimator dispersions reported in Table 1 provide a direct measure of the relative accuracy of each distance estimator at a true distance of 200 pc. In particular, we find that  $\hat{D}_{\text{ord}}$  is found to be accurate to  $\sim 7\%$  (at the  $1\sigma$  level) in the worst case examined where we have only 10 stars and with  $\sigma_\phi = 0.1$ ; this accuracy improves to slightly more than 4% with a sample of 30 stars. For  $\sigma_\phi = 0.05$  the accuracy of  $\hat{D}_{\text{ord}}$  improves from  $\sim 6\%$  to  $\sim 3.5\%$  as the sample size increases from  $n = 10$  to  $n = 30$ .

#### 4.5 Properties of Distance Estimators: Summary

The results of our Monte Carlo simulations - as summarised in Table 1 and illustrated in Figures 3, 4, 5, and 7 - demonstrate that the ordered distance estimator,  $\hat{D}_{\text{ord}}$ , is the best of the four distance estimators which we have considered in this paper. In all cases  $\hat{D}_{\text{ord}}$  is unbiased, and has the smallest dispersion. The naive estimator,  $\hat{D}_{\text{naive}}$ , would clearly be a poor choice on the other hand. This estimator is positively biased, resulting from the naive model for the distribution of  $D \sin i$  which is adopted in its definition. Moreover, the bias of  $\hat{D}_{\text{naive}}$  increases as the number of sampled stars increases. This property is particularly undesirable, since in the statistics literature one meets many biased estimators which are, nevertheless, *consistent* - meaning that their bias tends asymptotically to zero with increasing sample size.  $\hat{D}_{\text{naive}}$  instead displays precisely the opposite behaviour.

It is straightforward to compute the distributions of

these estimators at a range of different true cluster distances, and the same qualitative results are found in all cases, thus consolidating our choice of  $\hat{D}_{\text{ord}}$  as best cluster distance estimator.

Moreover, this Monte Carlo approach clearly provides a simple means of assigning errors to cluster distance estimates derived from real data - as we have already indicated in discussing the results of Table 1 above. Suppose, for example, that we infer an ordered distance estimate of  $\hat{D}_{\text{ord}} = \Delta$  from a given sample of real data. To calculate an error estimate we first generate a large number of random samples, each equal in size to the observed sample and assuming  $D_{\text{true}} = \Delta$ , and from a histogram of the distance estimates for these artificial samples we derive the distribution of  $\hat{D}_{\text{ord}}$  at this true distance. We then adopt the dispersion of this distribution as our error estimate for the cluster distance. In the same way we can use the distribution of  $\hat{D}_{\text{ord}}$  (or any other estimator) derived from our simulations to determine confidence intervals for the true cluster distance, given our estimated value.

This approach is straightforward to implement, but does suffer from one specific technical loophole: this concerns the fact that we generate the distribution of our estimator assuming  $D_{\text{true}} = \Delta$ , when it is not  $D_{\text{true}}$  but rather our *estimated* distance,  $\hat{D}_{\text{ord}}$ , which is equal to  $\Delta$ . A more rigorous method for assigning errors and determining confidence intervals which overcomes precisely this loophole does exist, and is described in detail in e.g. Hendry & Simmons (1990) or Mood & Graybill (1974). In the present context, however, we find that the confidence intervals derived by the simple method outlined above are essentially identical to those derived by the more rigorous approach, and so we do not describe the latter method here.

The smaller dispersion of  $\hat{D}_{\text{ord}}$ , as compared with  $\hat{D}_{\text{mean}}$  and  $\hat{D}_{\text{med}}$ , illustrates that by ordering the sampled  $D \sin i$  values one can frequently define a better cluster distance estimator;  $\hat{D}_{\text{ord}}$  being an example of one such estimator. There are a number of other methods by which one can use the properties of ordered samples to construct a cluster distance estimate. One appealing technique is to model the cumulative distribution function of  $D \sin i$  for a cluster at a given true distance, and then construct a sample cumulative distribution function to be compared with the model distribution. One would adopt as the cluster distance estimate the distance which ‘best fits’ the modelled  $D \sin i$  distribution to the sampled distribution. A suitable criterion for identifying the best fit might be, for example, the distance which minimises the Kolmogorov-Smirnov statistic for the two cumulative distributions. This statistic is frequently used in problems of this type and is particularly robust to the form of the underlying distribution (c.f. Kendall & Stuart (1963)). We have investigated the use of this method in the present context, however, and find that it gives no better (and frequently worse) results than using  $\hat{D}_{\text{ord}}$ . The robustness of this approach would be particularly useful, however, if one’s model for the distribution of  $D \sin i$  were in some way uncertain or ambiguous, and we will investigate the use of more robust estimation techniques in future work.

**Figure 6.** Distribution of the order statistics,  $\alpha_{(r)}$ , of the random variable  $\alpha$ , with pdf as given by Figure 1, from a sample size of  $n = 10$ . The  $r = 1$ ,  $r = 4$ ,  $r = 7$  and  $r = 10$  order statistics are shown.

**Table 1.** Mean value and dispersion of naive, mean, median and ordered cluster distance estimators, computed from 2000 simulations of cluster samples at a true distance of 200 pc. Results are given, in pc, for a sample size of  $n = 10$ , 20 and 30 stars, and for an angular diameter error dispersion of  $\sigma_\phi = 0.05$  and 0.1

$\sigma_\phi = 0.05$

$\hat{D}$	n = 10		n = 20		n = 30	
	mean	$\sigma_{\hat{D}}$	mean	$\sigma_{\hat{D}}$	mean	$\sigma_{\hat{D}}$
$\hat{D}_{naive}$	221.8	16.5	231.5	14.8	236.1	13.4
$\hat{D}_{mean}$	200.2	14.6	199.9	10.0	199.8	8.5
$\hat{D}_{med}$	199.4	16.2	199.8	11.6	199.8	9.8
$\hat{D}_{ord}$	199.9	12.2	199.9	8.5	199.9	7.1

$\sigma_\phi = 0.1$

$\hat{D}$	n = 10		n = 20		n = 30	
	mean	$\sigma_{\hat{D}}$	mean	$\sigma_{\hat{D}}$	mean	$\sigma_{\hat{D}}$
$\hat{D}_{naive}$	232.8	24.2	246.3	22.9	253.3	21.3
$\hat{D}_{mean}$	200.3	15.9	199.8	10.7	199.7	9.2
$\hat{D}_{med}$	200.3	17.6	200.3	12.5	200.5	10.7
$\hat{D}_{ord}$	200.1	14.6	200.1	9.8	200.2	8.4

**Figure 7.** Histograms of ordered cluster distance estimator distributions, derived from 2000 cluster samples at a true distance of 200 pc. Results are shown for a sample size of 10, 20 and 30 stars, and for an angular diameter error dispersion of  $\sigma_\phi = 0.05$  and 0.1

## 5 BAYESIAN CLUSTER DISTANCE ESTIMATES

In this section we briefly consider the estimation of the cluster distance as a problem in Bayesian inference. We will see that such a treatment is complementary to the analysis of distance estimators developed in section 4: indeed a Bayesian approach follows quite naturally from our chosen form for the ordered distance estimator.

The basic elements which comprise a Bayesian treatment of the problem can be summarised as follows. We begin by postulating a *prior* distribution for the parameter which we wish to estimate - i.e. the true cluster distance,  $D_{\text{true}}$ . This distribution is supposed to express our state of knowledge or ignorance about  $D_{\text{true}}$  before any data are obtained -

in our case the data being our ordered set of  $D \sin i$  values for the sampled stars in the cluster. Note that this postulate immediately represents a departure from our earlier view that  $D_{\text{true}}$  was a fixed, although unknown, parameter; we are now choosing to regard both  $D \sin i$  and  $D_{\text{true}}$  itself as random variables. Next we introduce a model which describes the probability of observing the data given the parameter value,  $D_{\text{true}}$  - in other words the expected distribution of ordered  $D \sin i$  values in a sample drawn from a cluster at a given true distance.

The essential idea of the Bayesian approach is to combine these two distributions - our prior distribution for  $D_{\text{true}}$ , and the  $D \sin i$  distribution given  $D_{\text{true}}$  - to derive a *posterior* distribution for  $D_{\text{true}}$ , which expresses our state of knowl-

edge of  $D_{\text{true}}$  after we have measured the values of  $D \sin i$ . The form of the posterior distribution is given by Bayes' theorem, and can be stated in the present context as:-

$$p(D_{\text{true}}|data) = \kappa p(data|D_{\text{true}})p(D_{\text{true}}) \quad (19)$$

Here  $p(D_{\text{true}})$  denotes our prior distribution for the true cluster distance, and  $\kappa$  is a normalisation constant which ensures that the posterior distribution integrates to 1.

The Bayesian formulation is not so far removed from the ideas which underpin our definition of  $\hat{D}_{\text{ord}}$  in Section 4. The concept introduced in equation (17) of a likelihood function,  $\lambda_{(r)}$ , for  $D_{\text{true}}$  given the measured value of  $D \sin i_{(r)}$ , has already alluded to the fact that we could regard  $D_{\text{true}}$  as a random variable. The Bayesian viewpoint extends this interpretation of the likelihood function to derive not simply a point estimate but rather a probability distribution for  $D_{\text{true}}$ , in the light of the observed data. In the same way as in section 4, then, we can regard the probability distribution,  $p(data|D_{\text{true}})$ , on the right hand side of equation (20) as a function of  $D_{\text{true}}$ : i.e. a likelihood function,  $L(D_{\text{true}}|data)$ , for  $D_{\text{true}}$  given the measured  $D \sin i$  values. The likelihood function clearly plays a crucial role in determining our posterior distribution: it is the function through which the observed data modifies our prior knowledge of  $D_{\text{true}}$ , and can therefore be thought of as representing the information about  $D_{\text{true}}$  which comes directly from the data.

In the present context  $L(D_{\text{true}}|data)$  is, therefore, given by the joint distribution of ordered  $D \sin i$  values expected in a cluster at true distance,  $D_{\text{true}}$ , viz:-

$$L(D_{\text{true}}|data) = p_{1...n}(D \sin i_{(1)}, \dots, D \sin i_{(n)}|D_{\text{true}}) \quad (20)$$

It follows, moreover, from equation (9) that we can rewrite this as:-

$$L(D_{\text{true}}|data) = p_{1...n}(\alpha_{(1)}, \dots, \alpha_{(n)}) \quad (21)$$

Where, of course,  $\alpha_{(r)} = \frac{D \sin i_{(r)}}{D_{\text{true}}}$ , for all  $r = 1, \dots, n$ , as before. A general expression for this joint distribution is given in David (1981) in terms of the density function and cumulative distribution of  $\alpha$ . It is important to note that this joint distribution will *not* in general be equal to  $\Lambda(D)$ , as defined in equation (18) above, however. In other words the joint likelihood function for all of the  $D \sin i$  order statistics is not in general equal to the product of the individual likelihood functions. This is because the sample  $D \sin i$  values are not independent of each other: the measured value of  $D \sin i_{(n)}$ , for example, must have a bearing on the distribution of  $D \sin i_{(n-1)}$ , since we now have the constraint that  $D \sin i_{(n-1)} \leq D \sin i_{(n)}$ , and so on for the smaller order statistics.

We now illustrate the application of this Bayesian approach to some typical cluster samples. Figure 8 shows the posterior distribution for  $D_{\text{true}}$  obtained from an ordered sample of  $D \sin i$  values, computed for sample sizes ranging between 10 and 30 stars. All samples were drawn from the simulated  $D \sin i$  distribution of a cluster at a distance of 200 pc, and with  $\sigma_{\phi} = 0.05$ . In each case the prior distribution for  $D_{\text{true}}$  was taken to be uniform within the range 150 to 250 pc.

We can see from Figure 8 that the Bayesian approach yields results which are broadly consistent with the estimator distributions derived from Monte Carlo sampling in Section 4. With a sample size of 10 stars, for example, we find

that the posterior distribution has a dispersion of  $\sim 10$  pc, which is consistent with the error estimates derived for  $\hat{D}_{\text{ord}}$  in Table 1. One can, of course, also derive Bayesian confidence intervals for  $D_{\text{true}}$  directly from the posterior distribution.

It is clear from Figure 8 that the posterior distribution becomes 'sharper' as the sample size increases - indicating a more accurate Bayesian estimation of the true distance from larger samples, as one would expect. Moreover, we find that there is little further reduction in the dispersion of the posterior when one's sample contains more than  $\sim 30$  stars - which is also analogous to the results of Section 4.

One need not carry out our Bayesian reconstruction using all  $n$  order statistics: one can use any single order statistic, or any subset of the full range of orders, with results which accord with the distributions of the order statistics used. The posterior distribution recovered from using only the smallest  $D \sin i$  value, for example, is considerably poorer than those shown in Figure 8: this merely reflects the relatively large dispersion of the  $\alpha_{(1)}$  distribution - as can be seen in Figure 6 above. In general we find that - for a given sample - the posterior of smallest dispersion is obtained by using the complete ordered sample of  $D \sin i$  values.

The choice of prior distribution in the application of Bayesian methods is clearly very important. The use of a non-uniform prior is the source of considerable controversy in the statistics literature, since it is often alleged that such a prior prejudices one's results by, as it were, forcing the data to say something different about  $D_{\text{true}}$  than the information which they in fact contain. In recognition of this point, a uniform prior seems to us to be a more appropriate choice. In this case it is solely the properties of the likelihood function which determine the form of the posterior distribution - and not some predisposed view of what the true cluster distance 'should' be.

Nevertheless, a powerful feature of the Bayesian approach is the fact that one can take the posterior distribution recovered from a given cluster sample and adopt this as a *new* prior, in order to determine an 'updated' posterior distribution in the light of new data - i.e. as  $D \sin i$  values are measured for additional stars in the cluster. We will investigate this extension to our analysis in future work.

## 6 CONCLUSIONS

In this paper we have presented a new method for determining the distance to nearby open clusters. The method is applicable to clusters containing fast-rotating late type stars, whose rotation periods have been measured from detecting the rotational modulation of surface inhomogeneities. The period of each star is then combined with its projected rotational velocity and an estimate of its angular diameter, inferred from the Barnes-Evans relation, to form an estimate of the projected cluster distance,  $D \sin i$ .

We have shown how one may then combine the set of  $D \sin i$  values inferred from each star in the cluster sample to form an 'ordered' distance estimator of the true cluster distance. We have investigated the properties of this estimator using Monte Carlo simulations of cluster samples, after careful modelling of the intrinsic scatter in the Barnes-Evans relation and the observational selection effects to which the

**Figure 8.** Examples of the posterior distribution for the true cluster distance, derived by applying the bayesian method described in the text to simulated cluster samples containing between 10 and 30 stars. In all cases the simulated samples were drawn from a cluster at a true distance of 200 pc

samples are subject. We have shown how one may apply this distance method to real data samples, in order to derive error estimates and confidence intervals for the true cluster distance. We have also demonstrated that this new method is amenable to a Bayesian analysis, and have again illustrated this approach using artificially generated cluster samples.

We have found that, for realistic models of the random variables, the accuracy of our distance method is between  $\sim 3$  and  $7\%$  (at the  $1\sigma$  level) - depending on the size of the cluster sample - which is comparable with the precision of distance estimates obtained by ZAMS fitting techniques. Since it is subject to a different set of systematic errors and model assumptions than ZAMS fitting - and in particular does not rely upon a zero-point calibration by a single cluster, usually the Hyades - these results indicate that our new method can play a useful and important role alongside more traditional cluster distance indicators in better determining the local distance scale.

In a forthcoming paper, currently in preparation, we will apply our new distance method to real data samples taken from the  $\alpha$  Per and Pleiades clusters, and explicitly compare the results of our method with those obtained by the ZAMS procedure. Other possible future applications of our method include a better calibration of the Barnes-Evans relation - and determination of the inclination distribution - for fast rotators in young open clusters, by comparing our results with the very accurate cluster distances soon to be provided by the Hipparcos satellite.

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